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# INVERSE BOUNDARY VALUE PROBLEM FOR A BOUSSINESQ TYPE NONLINEAR DIFFERENTIAL EQUATION

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Abstract. This article considers the questions of one value solvability of the inverse boundary value problem for a nonlinear Boussinesq type fourth-order integro-differential equation. The Fourier method of separation of variables is employed and is obtained the countable system of nonlinear integral equations (CSNIE). To prove the theorem of one value solvability of CSNIE is used the method of successive approximations. Further the article shows the convergence of the obtained Fourier series to the unknown function of boundary value problem and the stability of solution of the considering integro-differential equation with respect to restore Function. This paper advances the theory of partial differential equations with weak nonlinear hand-side.

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## 1. Problem Statement

Mathematical modeling of many processes occurring in the real world leads to the study of boundary value and inverse problems for equations of mathematical physics. Inverse and boundary value problems for partial differential equations by virtue of its importance in the application are one of the most important parts of the theory of differential equations. In the study of such problems of the theory of differential and integro-differential equations scientists are using different methods (see, for examples [1] – [11]).

In this paper we propose a method of studying the one-value solvability of the inverse problem for a nonlinear Boussinesq type fourth-order integro-differential equation. Boussinesq type differential equations model many natural phenomena and appear in many fields of sciences (see, for examples, [12]). For this reason, this type of equations was given a great importance in the works of many researchers.

We use the Fourier method of separation of variables. Application of the Fourier method of separation of variables can improve the quality of formulation of the considering problem and facilitates the processing procedure.

In the domain $\Omega$ is considered the partial differential equation of the following form

$$\text{(1)}\quad \frac{\partial^2 U(t,x)}{\partial t^2} - \frac{\partial^4 U(t,x)}{\partial t^2\,\partial x^2} - \mu(t)\frac{\partial^2 U(t,x)}{\partial x^2} = f\left(x, \int_0^T \int_0^l H(\theta,y)\,U(\theta,y)\,dy\,d\theta\right)$$

with boundary value

$$U(t,x)_{|t=0} = \varphi_1(x),\ U(t,x)_{|t=T} = \varphi_2(x),\ x \in \Omega_l, \tag{2}$$

$$U(t,x)_{|x=0} = U(t,x)_{|x=l} = 0,\ t \in \Omega_T \tag{3}$$

and additional condition

$$\frac{\partial}{\partial t} U(t,x)_{|\,t=T} = r(x),\ x \in \Omega_l, \tag{4}$$

where $f(x,\gamma) \in C(\Omega_l \times R)$, $\varphi_k(x) \in C^2(\Omega_l)$ , $\varphi_k(x)_{|x=0} = \varphi_k(x)_{|x=l} = 0$, $k = 1,\ 2$, $\varphi_2(x)$ is restore function, $\int\limits_0^T \int\limits_0^l |H(t,x)| dx\, dt < \infty$, $r(x) \in C^2(\Omega_l)$ , $\mu(t) \in C(\Omega_T)$ is small variable, $\Omega \equiv \Omega_T \times \Omega_l$, $\Omega_T \equiv [0,T]$, $\Omega_l \equiv [0,l]$, $0 < T < \infty$ , $0 < l < \infty$.

The pair of functions $\left\{U(t,x) \in C^{2,2}(\Omega),\ \varphi_2(x) \in C(\Omega_l)\right\}$ is called as a solution of the inverse problem (1)–(4), if it satisfies the equation (1) and the conditions (2)–(4).

## 2. Reduction of Main Unknown Function to a Countable System of Nonlinear Integral Equations

We find the solution of this problem in the following Fourier series:

$$U(t,x) = \sum_{n=1}^{\infty} u_n(t)\, \vartheta_n(x), \tag{5}$$

where functions $\vartheta_n(x)$ defined as eigenfunctions of the spectral problem $\vartheta''(x) + \lambda^2 \vartheta(x) = 0$, $\vartheta(x)_{|x=0} = \vartheta(x)_{|x=l} = 0$, $0 < \lambda$ and represent a complete system of eigenfunctions $\left\{\vartheta_n(x)\right\}_{n=1}^{\infty}$ in $L_2(D_l)$, while $\lambda_n$ are the corresponding eigenvalues.

We assume that

$$f\,(x,\gamma) = \sum_{n=1}^{\infty} f_n(\cdot)\, \vartheta_n(x), \tag{6}$$

where $f_n(\cdot) = \int\limits_0^l f\left(y, \int\limits_0^T \int\limits_0^l H(\theta,z) \sum\limits_{k=1}^{\infty} u_k(\theta)\, \vartheta_k(z) dz\, d\theta\right) \vartheta_n(y) dy$.

Substituting the series (5), (6) into the equation (1) and taking

$$\vartheta''_n(x) = -\lambda_n^2\, \vartheta_n(x)$$

into account, we obtain the following countable system of differential equations:

$$u''_n(t) + \tau_n\, \mu(t)\, u_n(t) = \frac{1}{1+\lambda_n^2} f_n(\cdot), \tag{7}$$

where $\tau_n = \dfrac{\lambda_n^2}{1+\lambda_n^2}$, $u_n(t) = \int\limits_0^l U(t,y)\, \vartheta_n(y) dy$.

The boundary value conditions (2) for the equation (7) we rewrite in the following form $u_n(0) = \varphi_{1n}$ , $u_n(T) = \varphi_{2n}$ , where $\varphi_{kn} = \int\limits_0^l \varphi_k(y)\, \vartheta_n(y) dy$, $k = 1,\ 2$. Then by integration with respect to a variable $t$ from (7) we derive

$$u_n(t) = h_n(t) - \int_0^t \overline{G}_n(t,s)\, u_n(s)\, ds + \frac{\zeta(t)}{1+\lambda_n^2} f_n(\cdot), \tag{8}$$

where $h_n(t) = \left(1 - \frac{t^2}{T^2}\right)\varphi_{1n} + \frac{t^2}{T^2}\varphi_{2n}$, $\overline{G}_n(t,s) = \tau_n\, \mu(s)\, G(t,s)$,

$\zeta(t) = \int\limits_0^t G(t,s)ds$ , $G(t,s) = \begin{cases} t(T-s), t<s, \\ s(T-t), s<t. \end{cases}$

The (8) we rewrite as follow

$$u_n(t) = Q_n(t) + \varphi_{2n} P(t) - \int_0^t \overline{G}_n(t,s)\, u_n(s)\, ds + f_n(\cdot)\, \Phi_n(t), \tag{9}$$

where $Q_n(t) = \left(1 - \dfrac{t^2}{T^2}\right)\varphi_{1n}$, $P(t) = \dfrac{t^2}{T^2}$, $\Phi_n(t) = \dfrac{\zeta(t)}{1+\lambda_n^2}$.

Substituting (9) into the Fourier series (5), we have the formal solution of boundary value problem (1)–(3)

$$U(t,x) = \sum_{n=1}^{\infty} \vartheta_n(x)\ \{Q_n(t) + \varphi_{2n} P(t) - \tag{10}$$

$$- \int_0^t \overline{G}_n(t,s)\, u_n(s) ds + f_n(\cdot)\, \Phi_n(t) \Bigg\}.$$

Using the condition (4), from (10) we have

$$r(x) = \sum_{n=1}^{\infty} \vartheta_n(x) \left\{ Q'_n(T) + \varphi_{2n} P'(T) - \int_0^T a_n(s)\, u_n(s)\, ds + \Phi'_n(T)\, f_n(\cdot) \right\},$$

where $Q'_n(T) = \left(1 - \dfrac{2}{T}\right)\varphi_{1n}$, $P'(T) = \dfrac{2}{T}$, $\Phi'_n(T) = \dfrac{1}{1+\lambda_n^2}\int\limits_0^T b(s)ds$,

$a_n(s) = \dfrac{\partial}{\partial t}\overline{G}_n(t,s)$, $b(s) = \dfrac{\partial}{\partial t} G(t,s)$.

Hence we find the formula to define the restore quantity

$$\varphi_{2n} = \chi_{1n} + \frac{T}{2}\int_0^T a_n(s)\, u_n(s)\, ds - \tag{11}$$

$$-\chi_{2n} \int_0^l f\left(y, \int_0^T \int_0^l H(\theta,z) \sum_{k=1}^{\infty} u_k(\theta)\, \vartheta_k(z)\, dz\, d\theta\right) \vartheta_n(y)\, dy,$$

where $\chi_{1n} = \dfrac{T}{2} r_n + \left(1 - \dfrac{T}{2}\right)\varphi_{1n}$, $\chi_{2n} = \dfrac{T}{2}\Phi'_n(T)$, $r_n = \int\limits_0^l r(y)\ \vartheta_n(y)\, dy$.

Substituting (11) into (9), for the main unknown function we derive

$$u_n(t) = \Im(u_n) \equiv g_{1n}(t) + \int_0^T R_n(t,s)\, u_n(s)\, ds + \tag{12}$$

$$+g_{2n}(t)\int\limits_0^l f\left(y,\int\limits_0^T\int\limits_0^l H(\theta,z)\sum_{k=1}^{\infty}u_k(\theta)\,\vartheta_k(z)\,dz\,d\theta\right)\vartheta_n(y)dy,$$

where $g_{1n}(t)=Q_n(t)+\chi_{1n}P(t)$, $g_{2n}(t)=\Phi_n(t)-\chi_{2n}P(t)$,

$$R_n(t,s)=\begin{cases}-\overline{G}_n(t,s)+\dfrac{T}{2}P(T)\,a_n(s),\ s<t,\\ \dfrac{T}{2}P(T)\,a_n(s),\ t<s.\end{cases}$$

## 3. One-value Solvability of CSNIE

The norm in the space $B_2(T)$ we take as follow

$$\|u(t)\|_{B_2(T)}=\sqrt{\sum_{n=1}^{\infty}\max_{t\in\Omega_T}\left|u_n(t)\right|^2}.$$

For function $g(x)\in L_2(\Omega_l)$ will be considered the following norm

$$\|g(x)\|_{L_2(\Omega_l)}=\sqrt{\int\limits_0^l\left|g(y)\right|^2dy}.$$

For the number sequence $\left\{\varphi_n\right\}_{n=1}^{\infty}\in\ell_2$ we use the following norm

$$\|\varphi\|_{\ell_2}=\sqrt{\sum_{n=1}^{\infty}\left|\varphi_n\right|^2}.$$

**Theorem 3.1.** *Let us assume that the following conditions are fulfilled:*

1). $\omega_k=\|g_k(t)\|_{B_2(T)}<\infty,\ k=1,2;\ \Delta=\int\limits_0^T\|R(t,s)\|_{B_2(T)}ds<\infty;$

2). $M_0=\|f(x,0)\|_{L_2(\Omega_l)}<\infty;\ M_1=\int\limits_0^T\int\limits_0^l\left|H(t,x)\right|dx\,dt<\infty;$

3). $\left|f(x,\gamma_1)-f(x,\gamma_2)\right|\le w(x)\left|\gamma_1-\gamma_2\right|;\ \delta_0=\|w(x)\|_{L_2(\Omega_l)}<\infty\ ;$

4). $\rho=\Delta+\delta_0\omega_2M_1M_2<1,$ *where* $M_2=\max_{x\in\Omega_l}\sqrt{\sum\limits_{n=1}^{\infty}\left|\vartheta_n(x)\right|^2}.$

*Then CSNIE* (12) *has a unique solution. This solution can be calculated by the method of successive approximation:*

$$\begin{cases}u_n^0(t)=0,\\ u_n^{j+1}(t)=\Im(u_n^j),\ j=1,2,\cdots.\end{cases}\tag{13}$$

*Proof.* Using the Hölder and Minkowski inequalities to the first differences, by virtue of conditions of theorem, from approximation (13) we can obtain the following estimate

$$\|u^1(t)-u^0(t)\|_{B_2(T)}\le\sqrt{\sum_{n=1}^{\infty}\max_{t\in\Omega_T}\left|g_{1n}(t)\right|^2}+\tag{14}$$

$$+\sqrt{\sum_{n=1}^{\infty}\max_{t\in\Omega_T}\left[\left|g_{2n}(t)\right|\int_0^l|f(y,0)|\cdot\left|\vartheta_n(y)\right|dy\right]^2}\le\omega_1+\omega_2 M_0.$$

Using the Hölder, Minkowski and Bessel inequalities to the arbitrary differences, from approximation (13) we derive the following estimate

$$\left\| u^{j+1}(t)-u^j(t)\right\|_{B_2(T)}\le \tag{15}$$

$$\le\sqrt{\sum_{n=1}^{\infty}\max_{t\in\Omega_T}\left[\int_0^T|R_n(t,s)|\cdot\left|u_n^j(s)-u_n^{j-1}(s)\right|ds\right]^2}+\sqrt{\sum_{n=1}^{\infty}\max_{t\in\Omega_T}\left|g_{2n}(t)\right|^2}\times$$

$$\times\sqrt{\sum_{n=1}^{\infty}\left[\int_0^l|w(y)|\int_0^T\int_0^l|H(\theta,z)|\sum_{k=1}^{\infty}\left|u_k^j(\theta)-u_k^{j-1}(\theta)\right|\cdot|\vartheta_k(z)|dz\,d\theta\right|\vartheta_n(y)\left|dy\right]^2}\le$$

$$\le\Delta\left\| u_n^j(t)-u_n^{j-1}(t)\right\|_{B_2(T)}+\omega_2 M_2\times$$

$$\times\sqrt{\sum_{n=1}^{\infty}\left[\int_0^l|w(y)|\int_0^T\int_0^l|H(\theta,z)|\,\| u^j(\theta)-u^{j-1}(\theta)\|_{B_2(T)}dz\,d\theta\right|\vartheta_n(y)\left|dy\right]^2}\le$$

$$\le\left\{\Delta+\omega_2 M_1 M_2\sqrt{\sum_{n=1}^{\infty}\left[\int_0^l|w(y)|\cdot|\vartheta_n(y)|\,dy\right]^2}\right\}\left\| u^j(t)-u^{j-1}(t)\right\|_{B_2(T)}\le$$

$$\le\rho\left\| u^j(t)-u^{j-1}(t)\right\|_{B_2(T)}.$$

By virtue of last condition of this theorem, from estimate (15) implies, that the operator in the right-hand side of (12) is compressing. From estimates (14) and (15) we conclude, that for the operator (12) there is exist a unique fixed point. Consequently, the CSNIE(12) has a unique solution $u(t)\in B_2(T)$. Moreover, there is true the following convergence rate

$$\left\| u^{j+1}(t)-u(t)\right\|_{B_2(T)}\le\frac{\rho^{j+1}}{1-\rho}\Big(\omega_1+\omega_2 M_0\Big).$$

The theorem was proved. □

We note that the set of integro-differential equations (1), for which is fulfilled the last condition of theorem, is not empty. Indeed, if we take as an example the function

$$H(t,x)=e^{-M_1 M_2\delta_0 t-\omega_2 x},$$

this condition takes the form

$$\rho=\Delta+\left(1-e^{-M_1 M_2\delta_0 T}\right)\left(1-e^{-\omega_2 l}\right)<1.$$

Hence we have that for $\Delta$, satisfying the inequality

$$\Delta<1-\left(1-e^{-M_1 M_2\delta_0 T}\right)\left(1-e^{-\omega_2 l}\right),$$

theorem 3.1 holds. $\Delta$ we can choose through small variable $\mu(t)$.

## 4. Solvability of Inverse Problem

**Theorem 4.1.** *Let us assume, that there are fulfilled the all conditions of the theorem* 3.1 *and* $u(t) \in B_2(T)$ *is the unique solution of CSNIE* (12). *If there is hold*

$$\nu_k = \left\| \chi_k \right\|_{\ell_2} < \infty, \; k = 1, 2, \tag{16}$$

*then inverse problem* (1)–(4) *is one-valued solvable.*

*Proof.* Substituting the solution of CSNIE (12) and iteration (13) into (5), we have

$$U(t,x) = \sum_{n=1}^{\infty} \Im(u_n)\, \vartheta_n(x) \equiv \sum_{n=1}^{\infty} \vartheta_n(x) \left\{ g_{1n}(t) + \int_0^T R_n(t,s)\, u_n(s)\, ds + \right. \tag{17}$$

$$\left. + g_{2n}(t) \int_0^l f\left( y, \int_0^T \int_0^l H(\theta, z) \sum_{k=1}^{\infty} u_k(\theta)\, \vartheta_k(z)\, dz\, d\theta \right) \vartheta_n(y) dy \right\},$$

$$U^{j+1}(t,x) = \sum_{n=1}^{\infty} \Im(u_n^j)\, \vartheta_n(x) \equiv \sum_{n=1}^{\infty} \vartheta_n(x) \left\{ g_{1n}(t) + \int_0^T R_n(t,s)\, u_n^j(s)\, ds + \right. \tag{18}$$

$$\left. + g_{2n}(t) \int_0^l f\left( y, \int_0^T \int_0^l H(\theta, z) \sum_{k=1}^{\infty} u_k^j(\theta)\, \vartheta_k(z)\, dz\, d\theta \right) \vartheta_n(y)\, dy \right\}.$$

We show that the sequence of functions (18) is convergence to the function (17) as $j \to \infty$. Let be $u(t) \in B_2(T)$ is unique solution of CSNIE (12) and

$$\left\| u^j(t) - u(t) \right\|_{B_2(T)} \le \frac{\varepsilon}{M_2},$$

where $0 < \varepsilon$ is small parameter. Then there is hold the following estimate

$$\left| U^j(t,x) - U(t,x) \right| \le \sum_{n=1}^{\infty} \left| u_n^j(t) - u_n(t) \right| \cdot \left| \vartheta_n(x) \right| \le$$

$$\le M_2 \left\| u^j(t) - u(t) \right\|_{B_2(T)} \le M_2 \frac{\varepsilon}{M_2} = \varepsilon.$$

Substituting the solution of CSNIE (12) into (11), we define the restore function

$$\varphi_2(x) = \sum_{n=1}^{\infty} \varphi_{2n}\, \vartheta_n(x) = \sum_{n=1}^{\infty} \vartheta_n(x) \left\{ \chi_{1n} + \frac{T}{2} \int_0^T a_n(s)\, u_n(s)\, ds - \right. \tag{19}$$

$$\left. - \chi_{2n} \int_0^l f\left( y, \int_0^T \int_0^l H(\theta, z) \sum_{k=1}^{\infty} u_k(\theta)\, \vartheta_k(z)\, dz\, d\theta \right) \vartheta_n(y)\, dy \right\}.$$

It is easy to verify that by the aid of condition (16) from (11) yields the estimate

$$\left\| \varphi_2 \right\|_{\ell_2} \le \nu_1 + \nu_2\, M_0 < \infty.$$

So for series (18) we have

$$\left|\varphi_2(x)\right| \le \sum_{n=1}^{\infty}\left|\varphi_{2n}\right|\cdot\left|\vartheta_n(x)\right| \le M_2\left\|\varphi_2\right\|_{\ell_2} \le M_2\left(\nu_1+\nu_2 M_0\right) < \infty.$$

Now we show the smoothness of solution of equation (1). Since because

$$\frac{\partial^2}{\partial t^2}R_n(t,s)\in C(\Omega_T),\ g_{kn}(t)\in C^2(\Omega_T), k=1,\,2,$$

from (12) we easily obtain that $\left\|u''(t)\right\|_{B_2(T)} < \infty$. So after applying Hölder inequality we have the following estimate

$$\left|\frac{\partial^2 U(t,x)}{\partial t^2}\right| \le \sum_{n=1}^{\infty}\left|u''_n(t)\right|\cdot\left|\vartheta_n(x)\right| \le M_2\left\|u''(t)\right\|_{B_2(T)} < \infty.$$

Differentiating (17) two times with respect to $x$ and taking $\vartheta''_n(x) = -\lambda_n^2\vartheta_n(x)$ into account we derive

$$\frac{\partial^2 U(t,x)}{\partial x^2} = \sum_{n=1}^{\infty} u_n(t)\,\vartheta''_n(x) = -\sum_{n=1}^{\infty}\lambda_n^2 u_n(t)\,\vartheta_n(x). \tag{20}$$

Twice integrating by parts the following integral

$$u_n(t) = \int_0^l U\,(t,y)\ \vartheta_n(y)\,dy,$$

we have

$$u_n(t) = -\frac{1}{\lambda_n^2}\int_0^l \frac{\partial^2 U(t,y)}{\partial y^2}\,\vartheta_n(y)\,dy. \tag{21}$$

Substituting (21) into (20) and using the Hölder and Bessel inequalities, finally we obtain

$$\left|-\sum_{n=1}^{\infty}\lambda_n^2 u_n(t)\,\vartheta_n(x)\right| = \left|\sum_{n=1}^{\infty}\int_0^l \frac{\partial^2 U(t,y)}{\partial y^2}\,\vartheta_n(y)dy\,\vartheta_n(x)\right| \le$$

$$\le \sqrt{\sum_{n=1}^{\infty}\left|\vartheta_n(x)\right|^2}\sqrt{\sum_{n=1}^{\infty}\left[\int_0^l\left|\frac{\partial^2 U(t,y)}{\partial y^2}\right|\cdot\left|\vartheta_n(y)\right|dy\right]^2} \le$$

$$\le M_2\left\|\frac{\partial^2 U(t,x)}{\partial x^2}\right\|_{L_2(\Omega_l)} < \infty.$$

The theorem was proved. □

Similarly, can be checked that the condition (16) of theorem 4.1 holds.

## 5. The Stability of Solution of Equation (1) with respect to Restore Function

**Theorem 5.1.** *Let us assume, that there are fulfilled all conditions of the theorem* 4.1. *If $u(t) \in B_2(T)$ is the unique solution of CSNIE* (12) *and $\varphi_2 \in \ell_2$ is the restore quantity, then the solution of equation* (1) *$U(t,x)$ is continuously dependent from restore function $\varphi_2(x)$.*

*Proof.* Let be $U_1(t,x)$ and $U_2(t,x)$ are different solutions of boundary value problem (1)–(3), corresponding two different values of restore function $\varphi_{21}(x)$ and $\varphi_{22}(x)$.

We assume that

$$\left\|\varphi_{21} - \varphi_{22}\right\|_{\ell_2} < \delta_1,\ 0 < \delta_1 = const.$$

Taking this estimate into account, by virtue of theorem conditions, from (9) we have

$$(22)\quad \left\| u_1(t) - u_2(t) \right\|_{B_2(T)} < \delta_1 + \max_{t\in\Omega_t} \int_0^T \left\| \overline{G}(t,s) \right\|_{B_2(T)} \left\| u_1(s) - u_2(s) \right\|_{B_2(T)} ds +$$

$$+\delta_0 M_2 \left\| \Phi(t) \right\|_{B_2(T)} \left\| u_1(t) - u_2(t) \right\|_{B_2(T)} \le \delta_1 +$$

$$+ \left(\Delta + \delta_0 M_2 \left\| \Phi(t) \right\|_{B_2(T)}\right) \left\| u_1(t) - u_2(t) \right\|_{B_2(T)}.$$

According to the theorem condition $1 - \Delta - \delta_0 M_2 \left\| \Phi(t) \right\|_{B_2(T)} > 0$. So from (22) we obtain

$$(23)\quad \left\| u_1(t) - u_2(t) \right\|_{B_2(T)} < \frac{\delta_1}{1 - \Delta - \delta_0 M_2 \left\| \Phi(t) \right\|_{B_2(T)}}.$$

Since because

$$\left| U_1(t,x) - U_2(t,x) \right| \le \sum_{n=1}^{\infty} \left| u_{1n}(t) - u_{2n}(t) \right| \cdot \left| \vartheta_n(x) \right| \le$$

$$\le M_2 \left\| u_1(t) - u_2(t) \right\|_{B_2(T)},$$

from (23) finally obtain the assertion of the theorem, if we set

$$\varepsilon = M_2 \frac{\delta_1}{1 - \Delta - \delta_0 M_2 \left\| \Phi(t) \right\|_{B_2(T)}}.$$

□

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